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Optimization of Structures with Multiple Design Variables Per Member

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Abstract

An optimality criterion method of design for structural frames composed of I-section members is presented. The I-section is defined by four independent geometric parameters which are taken as the member design variables. Constraints on minimum area, yielding, and local flange and web buckling are incorporated into the algorithm. The method is applied to a number of frame structures for which solutions and solution effort are known.

Contents

This paper deals with an optimality criterion approach for designing planar frames consisting of members with an I-cross-section describable by means of four parameters. Most present optimality criterion methods are based on one design variable per member or finite element (in Refs. 1-3, problems involving more than one design variable are considered). This one variable is usually area or moment of inertia. It is normally assumed that once this single variable is known, all other cross-sectional properties can be calculated using either exact or empirical relationships. In the problem considered herein, all four design variables per member are considered to be independent, and relationships among them are developed by means of the Kuhn-Tucker conditions.

Theory

Consider a structure consisting of I-beam members with cross sections as shown in Fig. 1. The height, width, and web and flange thicknesses are to be determined. Thus, for the ith member h_{1i}, h_{2i}, $(c_{1i}=t_{1i}/h_{1i})$, and $(c_{2i}=t_{2i}/h_{2i})$ are the four design variables (where the ratios t_{1i}/h_{1i} and t_{2i}/h_{2i} are used for convenience).

The design problem to be solved can be stated as: find the values of the design variables h_{1i}, h_{2i}, c_{1i}, and c_{2i} such that the volume of the structure:

$$V=\sum_{i=1}^{N}(c_{1i}h_{1i}^2+c_{2i}h_{2i}^2)L_i \tag{1}$$

is minimized, subject to the constraints

$$\sigma_i\leq\sigma_y \quad \sigma_i\leq\sigma_{fi} \quad \sigma_i\leq\sigma_{wi} \quad \sigma_{fi}\leq\sigma_{wi} \qquad i=1,\ldots,N \tag{2}$$

where σ_i is the maximum bending stress, σ_{fi} is the local buckling stress in the flange and σ_{wi} is the local buckling stress in the web for the ith member. Also, σ_y is the yield stress, L_i is the length, and N is the total number of members. The following expressions for the stresses are obtained from Ref. 3:

$$\sigma_i=\frac{6M_i}{h_{1i}(c_{1i}h_{1i}^2+6c_{2i}h_{2i}^2)} \quad \sigma_{fi}=1.54E_ic_{2i}^2 \quad \sigma_{wi}=21.7E_ic_{1i}^2 \tag{3}$$

where M_i is the maximum bending moment and E_i is the modulus of elasticity of the ith member.

The Lagrangian function for the above optimization problem can be written as:

$$\phi=V+\sum_i\lambda_{1i}(\sigma_i-\sigma_y)+\sum_i\lambda_{2i}(\sigma_i-\sigma_{fi})+\sum_i\lambda_{3i}(\sigma_i-\sigma_{wi})+\sum_i\lambda_{4i}(\sigma_{fi}-\sigma_{wi}) \tag{4}$$

where the λ are Lagrange multipliers. The optimality conditions are:

$$\frac{\partial\phi}{\partial h_{1i}}=\frac{\partial\phi}{\partial h_{2i}}=\frac{\partial\phi}{\partial c_{1i}}=\frac{\partial\phi}{\partial c_{2i}}=0 \qquad i=1,\ldots,N \tag{5}$$

It is concluded by contradiction (for details see Ref. 5) that the optimality conditions, Eq. (5), produce an optimal design for determinate structures in which:

$$\sigma_i=\sigma_{fi}=\sigma_{wi}=\sigma_y \tag{6}$$

Equations (5) and (6) reduce the optimization problem from four independent design variables to one variable per member. This variable is taken to be the height of the web h_{1i}. Therefore, the area A_i, moment of inertia I_i, and section modulus S_i of the ith section can be expressed as:

$$A_i=2c_{1i}h_{1i}^2 \quad I_i=c_{1i}h_{1i}^4/3 \quad S_i=2c_{1i}h_{1i}^3/3 \tag{7}$$

where $c_{1i}=\sqrt{\sigma_y/21.7E_i}$.

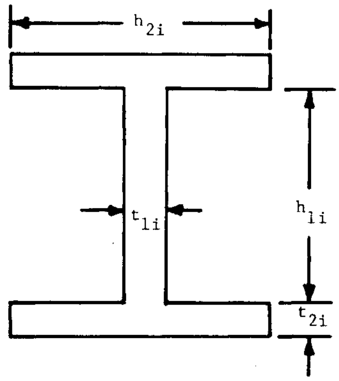

Fig. 1 I cross section.

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Table 1 Final design of a beam

Case No.	No. of analyses	No. of load conditions	Minimum area, m^2	Weights, N Present study	Ref. 4
1	19	1	—	30.41	30.43
2	19	2	—	32.62	32.64
3	26	1	—	21.07	21.07
4	23	1	129×10^{-6}	22.53	—

Table 2 Final design of 70-member frame (case 1)[a]

Member No.	Area, m^2	Member No.	Area, m^2	Member No.	Area, m^2	Member No.	Area, m^2
1	.07553	19	.01430	37	.01984	55	.01309
2	.05425	20	.01715	38	.02033	56	.01224
3	.03543	21	.01618	39	.01617	57	.01548
4	.02667	22	.02001	40	.01788	58	.01368
5	.02258	23	.01630	41	.01485	59	.01985
6	.01832	24	.02050	42	.01637	60	.01421
7	.01288	25	.01352	43	.01319	61	.02470
8	.00843	26	.02271	44	.01389	62	.01445
9	.00504	27	.02497	45	.01120	63	.03064
10	.00323	28	.00323	46	.01546	64	.02158
11	.00940	29	.00323	47	.01452	65	.04045
12	.01161	30	.00323	48	.00407	66	.02341
13	.00750	31	.00323	49	.00323	67	.06089
14	.00794	32	.00323	50	.00323	68	.00323
15	.00950	33	.00323	51	.00323	69	.08195
16	.01191	34	.02111	52	.00425	70	.00323
17	.01204	35	.01697	53	.00529		
18	.01504	36	.02544	54	.01406		

[a]Total volume (m^3) = 3.0445 for case 1; no. of analyses = 32.

In order to obtain an optimal design, the following iterative equations are used based on a fully stressed assumption (iterating on I_i instead of h_{1i}):

$$(I_i)_{\nu+1} = (\sigma_i/\sigma_y)^{\eta}_{\nu} (I_i)_{\nu} \qquad \text{if } \sigma_i \leq \sigma_y \tag{8}$$

$$(I_i)_{\nu+1} = (\sigma_i/0.98\sigma_y)^{1.5}_{\nu} (I_i)_{\nu} \qquad \text{if } \sigma_i > \sigma_y \tag{9}$$

where ν is an iteration counter and η is a relaxation parameter. The value of η is started at 0.8 and is reduced during the design process as the design converges to the optimum. The application of η is similar to move limits used in mathematical programming methods. Equation (9) is used to bring all those design members, which have stress σ_i greater than their maximum specified value, quickly into the feasible region. The stress σ_i in Eqs. (8) and (9) can be taken as the bending stress alone or a combination of bending and axial stress without difficulty. A minimum area limit can also be imposed easily during the design process.

Results

Equations (8) and (9) were used to design several frame structures. Two of these examples are presented here to show the efficiency and the effectiveness of the method.

1) This example, designed in Ref. 4, is a simply supported beam subjected to concentrated moments. The loading and other data are given in Ref. 4. Four cases are considered. Case 1 has one load condition and each span is taken as one finite element; case 2 has two load conditions using the same number of finite elements; case 3 has one load condition and the entire beam is divided into 12 finite elements of equal length; case 4 is essentially case 3 except a minimum area limit of 129×10^{-6} m^2 is imposed on each element. The optimal designs obtained for each case are shown in Table 1. The designs are in excellent agreement with those of Ref. 4.

2) This example is a 70-member steel frame shown in Fig. 2. A minimum area limit of 3.22×10^{-3} m^2 is imposed on

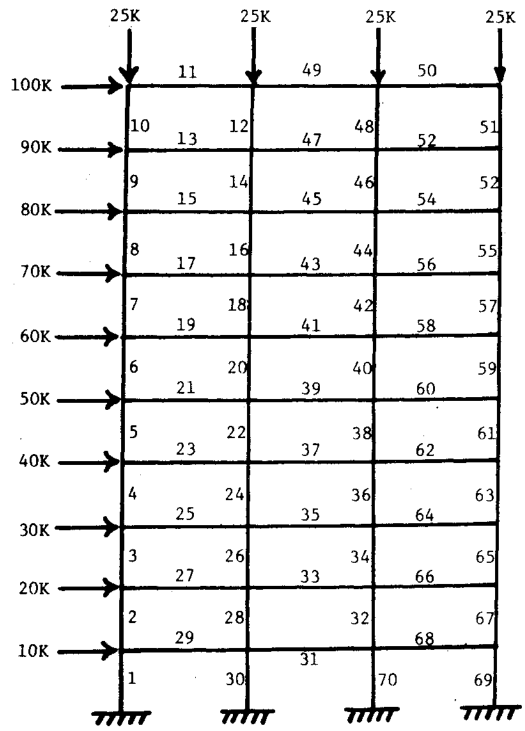

Fig. 2 The 70-member frame. (Each member length is 2.54 m) (1K = 4448 N.)

each member. One load condition is considered and the maximum stress limit is ±137.9 MPa. Two cases are considered. Case 1 calculates the maximum stress in each member using $\sigma_i = (|M|/S) + (|P|/A)$ (combined bending and axial) and, in case 2, $\sigma_i = |M|/S$ (bending stress alone). The starting design in the optimization was uniform 71.612×10^{-3} m^2. The final design for case 1 is given in Table 2. At the optimum, 12 members are at their lower bounds. The final volumes for cases 1 and 2 are 3.0445 and 2.4292 m^3, respectively. This volume for case 1 is 25% higher than for case 2. Therefore, the design obtained in case 2 is not satisfactory in this example, because the axial stresses in the members make a substantial contribution to the total stress.

An attempt was made to solve this problem in case 2 using the iterative technique of Nelson and Felton[4] starting from the same initial design. However, the method oscillated and never converged to the optimal design.

Conclusions

An optimality criterion method for the design of frames with I-section members, each described by four design variables, was presented. The optimality criterion provided relations among three of the variables and a recursive relationship for the fourth. The procedure is exact for statically determinate frames. It obtained an optimal design in all cases for statically indeterminate problems in from 15 to 40 iterations.

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